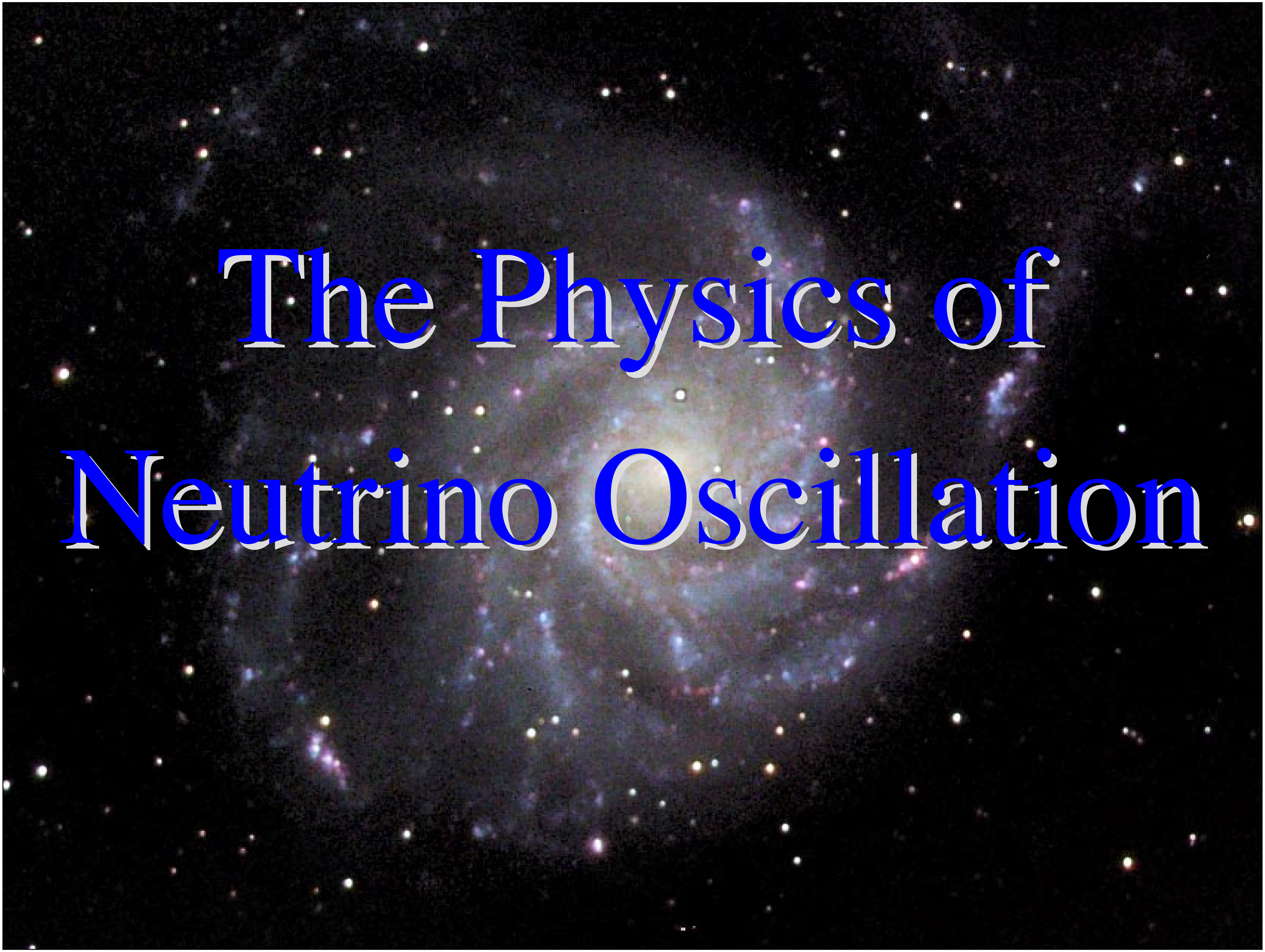
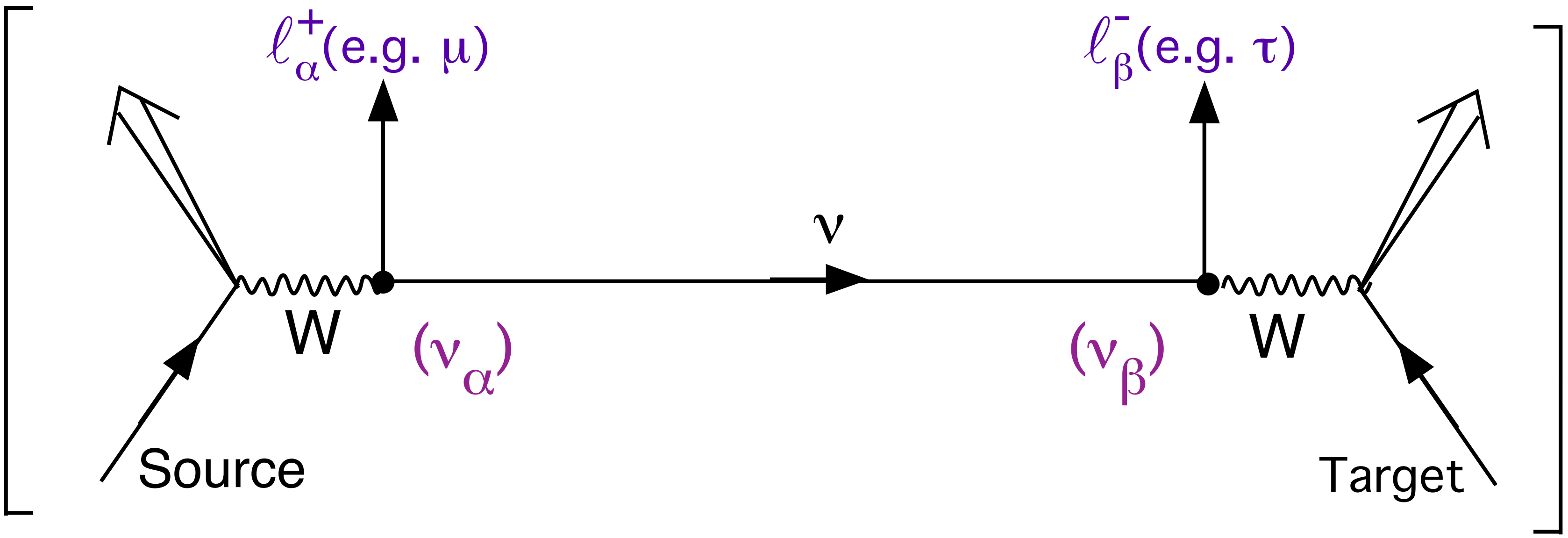
The Physics of Neutrino Oscillation



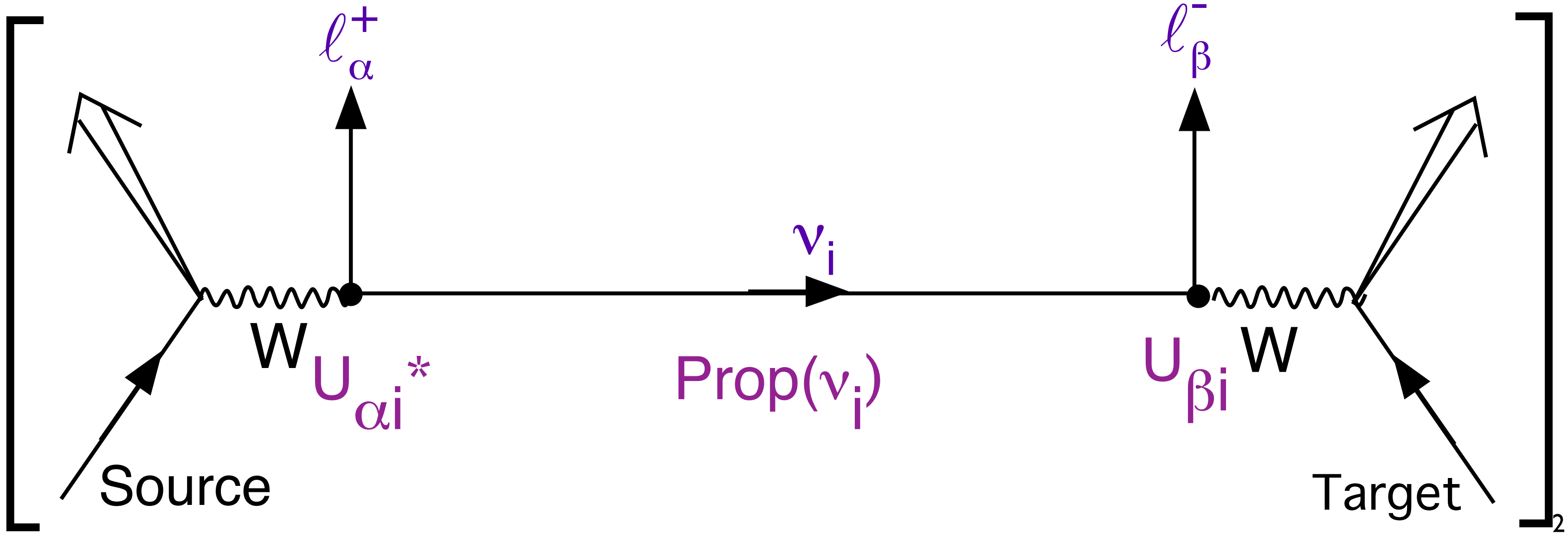
Neutrino Flavor Change (Oscillation) in Vacuum

(Approach of B.K. & Stodolsky)

Amp



$= \sum_i$ Amp



$|\ldots|^2$

Amp $[\nu_\alpha \rightarrow \nu_\beta] = \Sigma U_{\alpha i}^* \, \mathrm{Prop}(\nu_i) U_{\beta i}$

What is Propagator $(\nu_i) \equiv \mathrm{Prop}(\nu_i)$?

In the ν_i rest frame, where the proper time is τ_i,

$$i\frac{\partial}{\partial\tau_i}|\nu_i(\tau_i)>= m_i|\nu_i(\tau_i)> \quad .$$

Thus,

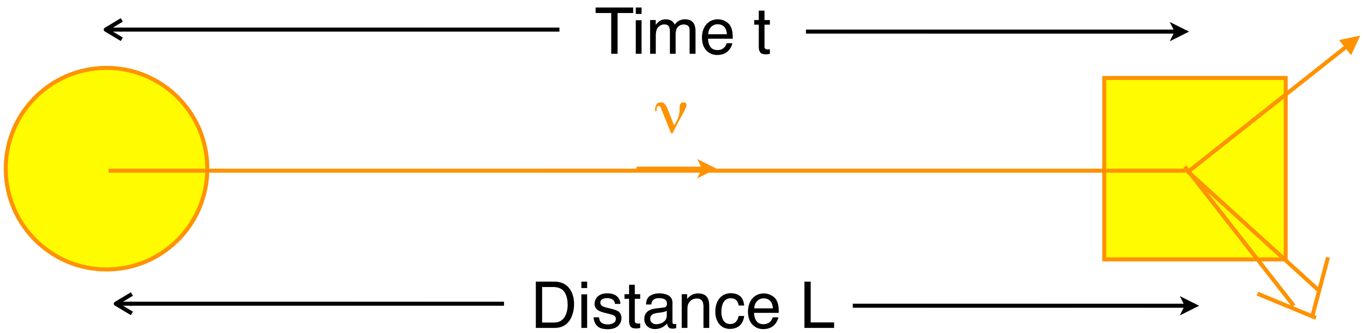
$$|\nu_i(\tau_i)>= e^{-im_i\tau_i}|\nu_i(0)> \quad .$$

Then, the amplitude for propagation for time τ_i

is —

$$\mathrm{Prop}(\nu_i) \equiv< \nu_i(0)|\nu_i(\tau_i)>= e^{-im_i\tau_i} \quad .$$

In the laboratory frame —



The experimenter chooses L and t.

They are common to all components of the beam.

For each ν_i, by Lorentz invariance,

$$(E_i, p_i) \times (t, L) = m_i \tau_i = E_i t - p_i L \ .$$

Neutrino sources are ~ constant in time.

Averaged over time, the

$e^{-iE_1t} - e^{-iE_2t}$ interference

is —

$$< e^{-i(E_1-E_2)t} >_t = 0$$

unless $E_2 = E_1$.

Only neutrino mass eigenstates with a common energy E are coherent.

(Stodolsky)

For each mass eigenstate ,

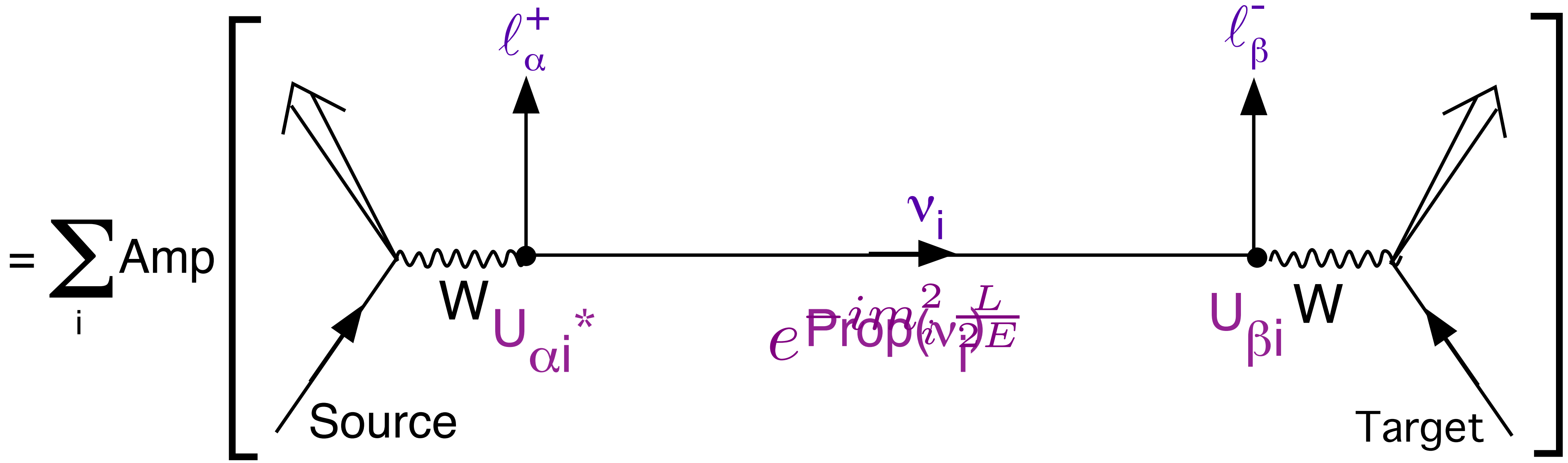
$$p_i = \sqrt{E^2 - m_i^2} \cong E - \frac{m_i^2}{2E} \quad .$$

Then the phase in the ν_i propagator $exp[-im_i\tau_i]$ is

$$m_i\tau_i = E_i t - p_i L \cong Et - (E - m_i^2/2E)L$$

$$= \underbrace{E(t - L)}_{\text{Irrelevant overall phase}} + m_i^2 L/2E \quad .$$

Amp $[\nu_\alpha \to \nu_\beta]$



$$= \sum_i U^*_{\alpha i} e^{-im_i^2 \frac{L}{2E}} U_{\beta i}$$

Probability for Neutrino Oscillation in Vacuum

$$P(\nu_\alpha \to \nu_\beta) = |\mathrm{Amp}(\nu_\alpha \to \nu_\beta)|^2 =$$

$$= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$+2 \sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

$$\text{where } \Delta m^2_{ij} \equiv m^2_i - m^2_j$$

For Antineutrinos –

We assume the world is CPT invariant.

Our formalism assumes this.

$$P(\overline{\nu_\alpha} \rightarrow \overline{\nu_\beta}) \overset{CPT}{=} P(\nu_\beta \rightarrow \nu_\alpha) = P(\nu_\alpha \rightarrow \nu_\beta; U \rightarrow U^*)$$

Thus,

$$P(\overset{(-)}{\nu}_\alpha \rightarrow \overset{(-)}{\nu}_\beta) =$$

$$= \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}) \sin^2(\Delta m^2_{ij}\frac{L}{4E})$$

$$\underset{(-)}{+}2\sum_{i>j} \Im(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}) \sin(\Delta m^2_{ij}\frac{L}{2E})$$

A complex U would lead to the CP violation

$P(\overline{\nu_\alpha} \rightarrow \overline{\nu_\beta}) \neq P(\nu_\alpha \rightarrow \nu_\beta)$.

Must we assume all mass eigenstates have the same *E*?

No, we can take entanglement into account, and use energy conservation.

The oscillation probabilities
are still the same.

B.K.: arXiv:1110.3047
ICL seminar two days ago