Integrals at NLO for the LHC

Gavin Cullen

The University of Edinburgh

15th May 2009

LHC

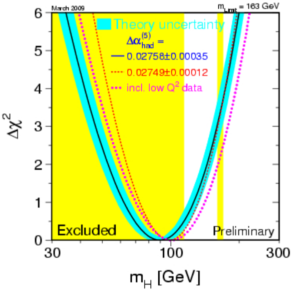
What are we looking for?

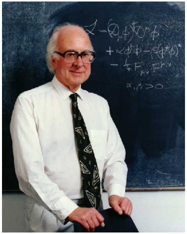
- New
  - BSM at 1 TeV? Supersymmetry, Extra dimensions...

LHC

What are we looking for?

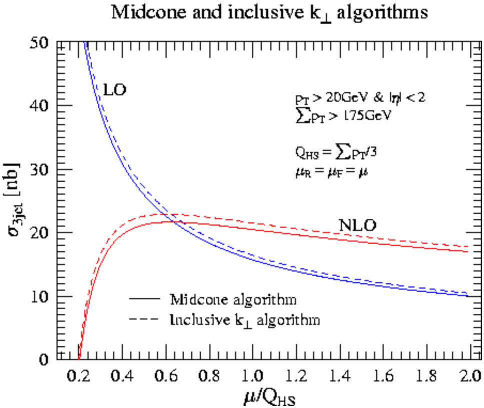
- New
  - BSM at 1 TeV? Supersymmetry, Extra dimensions...
- ...and old
  - Electroweak symmetry *is* broken: we have massive W, Z bosons
  - Where is the Higgs?
    - $m_h \geq 114.4 GeV$





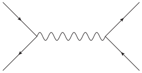
Why NLO?

Reduce sensitivity to choice of scale in cross section.



Next to Leading Order

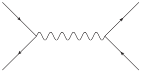
$$\sigma_{NLO} = \int_n d\sigma^{LO}$$



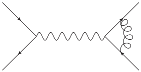
- Tree level

Next to Leading Order

$$\sigma_{NLO} = \int_n d\sigma^{LO}$$



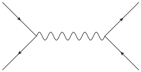
$$+ \int_n \left(d\sigma^V \qquad\qquad \right)$$



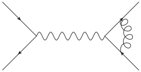
- Tree level
- Virtual corrections

Next to Leading Order

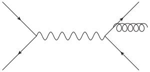
$$\sigma_{NLO} = \int_n d\sigma^{LO}$$



$$+ \int_n \left(d\sigma^{V} \qquad \right)$$



$$+ \int_{n+1} \left(d\sigma^{R} \qquad \right)$$

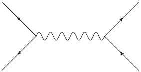


- Tree level
- Virtual corrections
- Real emissions

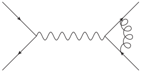
- Sum is not divergent!

Next to Leading Order

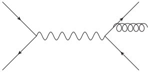
$$\sigma_{NLO} = \int_n d\sigma^{LO}$$



$$+ \int_n \left(d\sigma^V + \int_1 d\sigma^A \right)$$



$$+ \int_{n+1} \left(d\sigma^R - d\sigma^A \right)$$



- Tree level
- Virtual corrections
- Real emissions
- Subtraction terms for soft and collinear singularities

- Sum is not divergent!
- Subtract poles in ϵ

Efficient Calculations at NLO

Need to know:

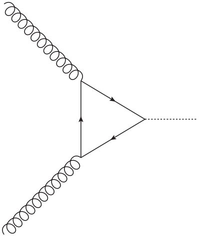
- Color Decomposition
  - Split into color and Lorentz part
- Spinor Helicity Formalism
  - Using Chiral projection operator $\Pi_\pm = \frac{1}{2}(1 \pm \gamma_5)$

$$\begin{aligned} A &= \overline{u}(p_2)(-ie\gamma^\mu)u(p_1) \\ &= -ie\overline{u}(p_2)(\Pi_+ + \Pi_-)\gamma^\mu(\Pi_+ + \Pi_-)u(p_1) \\ &= -ie\left\langle 2^- \middle| \gamma^\mu \middle| 1^- \right\rangle - ie\left\langle 2^+ \middle| \gamma^\mu \middle| 1^+ \right\rangle \\ &= A^{--} + A^{++} \end{aligned}$$

  - Compact representation of amplitudes
  - No interference terms between subamplitudes
  - Can be related to one another by parity transformations
- Loop Integrals

Loop Integrals

Consider the following example:

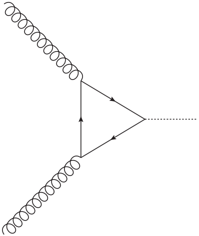


$$A \propto (-igT^A)(-igT^B)\epsilon^A_\alpha(p_1)\epsilon^B_\beta(p_2)$$
$$\int \frac{d^n k}{(2\pi)^n} \frac{i(-\not{p}_2 + \not{k})\gamma^\beta i(\not{p}_1 + \not{k})\gamma^\alpha i(\not{k} + \not{m})\gamma^\mu}{((-p_2 + k)^2 - m^2)((p_1 + k)^2 - m^2)(k^2 - m^2)}$$

- Color information
- Lorentz and kinematic structure

Loop Integrals

Consider the following example:

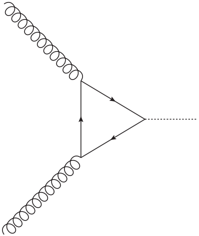


$$A \propto (-ig T^A)(-ig T^B)\epsilon_\alpha^A(p_1)\epsilon_\beta^B(p_2)$$
$$\int \frac{d^n k}{(2\pi)^n} \frac{i(-\not{p}_2 + \not{k})\gamma^\beta i(\not{p}_1 + \not{k})\gamma^\alpha i(\not{k} + \not{m})\gamma^\mu}{((-p_2 + k)^2 - m^2)((p_1 + k)^2 - m^2)(k^2 - m^2)}$$

- Color information
- Lorentz and kinematic structure

Loop Integrals

Consider the following example:

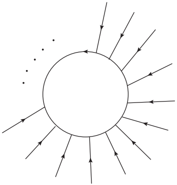


$$A \propto (-igT^A)(-igT^B)\epsilon_\alpha^A(p_1)\epsilon_\beta^B(p_2) \int \frac{d^nk}{(2\pi)^n} \frac{i(-\not{p}_2 + \not{k})\gamma^\beta i(\not{p}_1 + \not{k})\gamma^\alpha i(\not{k} + \not{m})\gamma^\mu}{((-p_2+k)^2 - m^2)((p_1+k)^2 - m^2)(k^2 - m^2)}$$

- Color information
- Lorentz and kinematic structure

Tensor Integrals

We arrive at integrals of the form



$$I_N^{n,\alpha\beta\gamma\ldots} = \int \frac{d^n k}{(2\pi)^n} \frac{k^\alpha k^\beta k^\gamma \ldots}{((k+p_1^2)^2 - m_1^2)((k+p_1+p_2)^2 - m_2^2)\ldots(k^2 - m_N^2)}$$

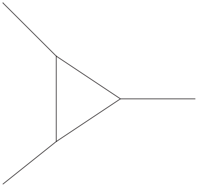
Daunting!

- We can disentangle the Lorentz structure from the integral
- We can reduce this integral to a basis set of scalar integrals

Passarino-Veltmann Reduction

A Simple Example

"simple" = massless propagators, triangle



$$I_3^{n,\mu} = \int \frac{d^n k}{(2\pi)^n} \frac{k^\mu}{k^2 (k+p_1)^2 (k_1 + p_1 + p_2)^2} \equiv p_1^\mu C_1 + p_2^\mu C_2$$

Project out C1,C2

Passarino-Veltmann Reduction

We arrive at

$$C_1 = \frac{1}{2detG}\left[I_2(p_1+p_2)(p_2^2+p_1\cdot p_2) + I_2(p_2)(-p_2^2-p_1\cdot p_2)\right.$$
$$\left. + I_3(p_1, p_1+p_2)(-p_1^2p_2^2 + 2(p_1\cdot p_2)^2 - p_2^2(p_1\cdot p_2))\right]$$

where $detG = p_1^2p_2^2 - (p_1\cdot p_2)^2$

- Can generalise to higher rank $I_3^{n,\mu\nu..}$
- In fact we can express every one loop diagram in terms of scalar integrals: $I_4^n, I_3^n, I_2^n, I_1^n$

Ideal for an automated approach to loop integrals?

Problems with Implementation

- G_{ij}
  - Coefficents contain inverse Gram Determinants : $G_{ij} = p_i \cdot p_j$
  - $detG \to 0$ in certain kinematical regions. Can't invert!
  - Can be worked around but requires care
  - Poses a problem to any automated approach
- Large intermediate expressions
  - Complexity of calculation grows like N!
  - Large cancellation between terms to give final answer
  - More efficient methods? (Unitarity based methods?)
  - Heated debate: evolve or die?

Solution:

Golem95 (Binoth, Guillet, Heinrich, Pilon, Reiter)

- Avoids inverse Gram Determinants by choosing a different set of basis integrals
- Integrals are defined in a non-standard way to be symmetric under shifts in loop momentum
- In singular regions code moves to numerical evaluation of basis integrals
- Massive internal propagators currently being implemented