The Little Hierarchy Problem in Warped Extra Dimensions

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Warped Extra Dimensions

- While the standard model is, experimentally, an immensely successful theory it is an effective theory and suffers from certain conceptual problems.
- One such problem is the Hierarchy problem, in which loop corrections to the Higgs Mass are some 30 orders of magnitude greater than the bare Higgs mass.
- One possible resolution to the Hierarchy problem, proposed by Randall and Sundrum (hep-ph/9905221), is to localize the Higgs at one end of a warped extra dimension.
- The effective Planck Mass is then suppressed down from a larger fundamental scale.

The Little Hierarchy Problem

- However despite extra dimensions being an integral part of so much BSM physics (eg. string theory), we quite clearly only see four!
- Observational constraints on extra dimensional models seem to force the scale of new physics to be much larger than that of the EW scale. This essentially the little hierarchy problem.
- Here I will look at the constraints on generic warped extra dimensional models from electoweak observables.
- Will the LHC see a Kaluza Klein Particle?

A brief history of EW analysis of Warped Extra Dimensions

- Csàki, Erlich and Terning consider the Higgs and fermions localized to the IR brane while the gauge field are allowed to propagate into the bulk. Compute Peskin Takeuchi parameters S, T and U. Find that a large contribution to T ($\sim$ corrections to gauge boson mass.) forces KK scale $>$ 11 TeV. (hep-ph/0203034)
  Lightest KK mass $M_{KK} \gtrsim 27$ TeV.
- Huber, Lee and Shafi place fermions and gauge field in the bulk and find you can lower constraint to $M_{KK} \gtrsim 10$ TeV (hep-ph/0111465).
- Carena, Pontón, Tait and Wagner allow bulk gauge and fermion field but also include large gauge kinetic terms on the branes. This brings $M_{KK} \gtrsim 5$ TeV but questionable physical motivation (hep-ph/0212307).

- Most popular approach proposed by Agashe, Delgado, May and Sundrum is to protect the T parameter with a custodial $(SU(2)_L \times SU(2)_R)$ bulk gauge symmetry. (hep-ph/0308036) This implies $M_{KK} \gtrsim 6$ TeV.

- Delgado and Falkowski considered a general 5D warped geometry and demonstrated that the large T parameter was proportional to an integral over the warp factor. (hep-ph/0702234);

The Strategy

- Rather than computing oblique corrections we directly compute the tree level corrections to individual EW observables.
- Carrying out the calculation with a general metric initially before specifying to first the Randall and Sundrum model and then a class of 5d deformed conifolds.
- By carrying out the KK decompositions before spontaneous symmetry breaking it is numerically simpler to take into account more of the the tower of KK modes.

The KK decomposition

Working in the general warped background;

$$ds^2 = a^2(r)\eta_{\mu\nu}dx^\mu dx^\nu - b^2(r)dr^2, \tag{1}$$

with bulk gauge and fermion field and a Higgs localized towards the 'IR' brane,

$$\boxed{\begin{aligned} S = \int d^5x\sqrt{-G}\Big\{ -\frac{1}{4}A^a_{MN}A^{MN\ a} - \frac{1}{4}B_{MN}B^{MN} \\ +\sum_j \bar{\psi}_j\left(i\gamma^N\nabla_N - M\right)\psi_j + \frac{\delta(r-r_{IR})}{\sqrt{G_{55}}}\Big[|D_\mu\Phi|^2 + V(\Phi)\Big]\Big\}. \end{aligned}}$$

Then decomposing the gauge and fermion fields (and working in R_ξ gauge i.e. $A_5 = 0$)

$$A_\mu = \sum_n f_A^{(n)}(r)A_\mu^{(n)}(x^\mu) \qquad \psi_{L,R} = \sum_n a^{-2}f_{L,R}^{(n)}(r)\psi_{L,R}^{(n)}(x^\mu) \tag{2}$$

where $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$.

The KK decomposition is chosen such that

$$\int dr\ b\ f_A^{(n)} f_A^{(m)} = \delta_{nm} \qquad \int dr\ \frac{b}{a}\ f_{L,R}^{(n)} f_{L,R}^{(m)} = \delta_{nm}. \tag{3}$$

Hence the wave functions will be solutions of;

$$f_A'' + \frac{(a^2 b^{-1})'}{a^2 b^{-1}} f_A' + \frac{b^2}{a^2} m_n^2 f_A = 0 \tag{4}$$

$$f_R' + b\ M\ f_R = \frac{b}{a} f_L m_n \tag{5}$$

$$-f_L' + b\ M\ f_L = \frac{b}{a} f_R m_n \tag{6}$$

The 4D effective action

The four dimensional effective action can then be obtained by integrating over the 5th dimension.

$$\boxed{\begin{gathered} S = \sum_n \int d^4x \Big[-\frac{1}{4} A^{(n)a}_{\mu\nu} A^{(n)a\mu\nu} - \frac{1}{4} B^{(n)}_{\mu\nu} B^{\mu\nu(n)} + \frac{1}{2} m_n^2 A^{(n)}_\mu A^{\mu(n)} + \\ + \frac{1}{2} m_n^2 B^{(n)}_\mu B^{(n)\mu} + \bar{\psi}^{(n)} \left(i\gamma^\mu D^{(n)}_\mu - m_n \right) \psi^{(n)} + |D_\mu \Phi|^2 + V(\Phi) \Big] \end{gathered}}$$

where now

$$D^{(n)}_\mu = \partial_\mu + \sum_m \Big[-ig_5 \left(\int dr \frac{b}{a} f^{(n)}_L f^{(m)}_A f^{(n)}_L \right) A^{(m)}_\mu - ig_5' \left(\int dr \frac{b}{a} f^{(n)}_L f^{(m)}_B f^{(n)}_L \right) B^{(m)}_\mu \Big] \tag{7}$$

So now 'normal' phenomenology can be done. Note that after SSB there will be a mixing between the higher KK gauge modes and the gauge mass matrix will get off diagonal terms.

Fixing the Input Parameters

The measure of a theory lies in finding input parameters which can generate all observable quantities. To tree level EW sector governed by 3 parameters, g, g' and v which we fix by comparison with 3 most precisely observed quantities, G_f, M_Z and α, given by;

$$\sqrt{4\pi\alpha} = \frac{g_5 g_5'}{\sqrt{g_5^2 + g_5'^{\,2}}} f_\psi^{(0)} \tag{8}$$

$$4\sqrt{2} G_f = g_5^2 \sum_{n\ m} f_\psi^{(m)} \left(M_W\right)_{mn}^{-1} f_\psi^{(n)} \tag{9}$$

$$M_Z^2 = \text{diag}\left(m_n^2 \delta_{mn} + \frac{(g_5^2 + g_5'^2)v^2}{4} f_A^{(n)} f_A^{(m)} \right)_{00} \tag{10}$$

Where $f_\psi^{(n)} \equiv \int dr \frac{b}{a} f_L^{(0)} f_A^{(n)} f_L^{(0)}$.
Once the input parameters are fixed you can compute EW observables and compare with deviations between SM and observed values to arrive at EW constraints on model.

The effect of including 'more' of the KK tower

Even though considering tree level corrections to LEP I (Z pole) Data you still need to consider more than just the zero mode.

Constraint from EWO	Size of Mass Matrix				
(TeV)	2×2	3×3	4×4	5×5	10×10
M_W	14.06	15.38	15.88	16.13	16.61
Γ_Z	14.24	15.56	16.08	16.34	16.82
$\Gamma(had)$	11.44	12.51	12.92	13.14	13.52
$\Gamma(inv)$	9.77	10.67	11.03	11.21	11.53
$\Gamma(l^+l^-)$	15.15	16.55	17.11	17.40	17.88
R_e	5.69	6.22	6.42	6.53	6.71
A_e	20.08	21.89	22.66	23.03	23.70
s_Z^2	26.10	28.51	29.41	29.93	30.74
min KK scale (R'/R^2)	10.7	11.7	12.0	12.2	12.5

Table: The minimum unperturbed KK mass (m_1) that satisfies the experimental constraints of a give EW observable with a 95% CL for $\Omega = 10^{15}$ and the fermions are localized on the IR brane in a RS scenario. The bottom row is the KK scale ($\approx m_1/2.45$) arising from the tightest constraint i.e. s_Z^2.

Randall and Sundrum with bulk fermions

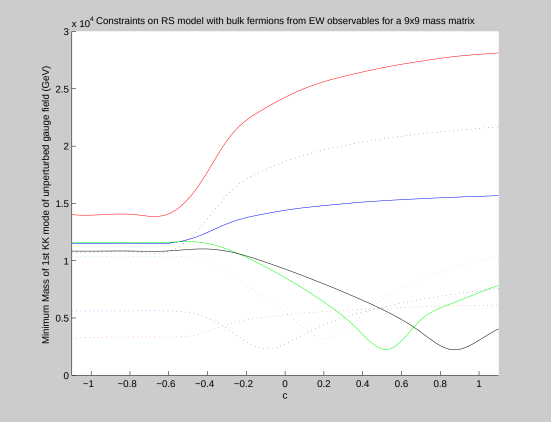


Figure: The lower bound on the KK Gauge mass from the EWO's; S_Z^2 (Red line), M_W (blue line), Γ_Z (green line), Γ_{had} (black line), R_e (blue dots), Γ_{inv} (red dots), $\Gamma_{l^+l^-}$ (green dots) and A_e (black dots). On the x axis is 5D Dirac Mass, $c = \frac{M}{R}$ with $c < -0.5 (> -0.5)$ meaning the fermions are localized towards the UV (IR) brane.

The Mass Gap Metric

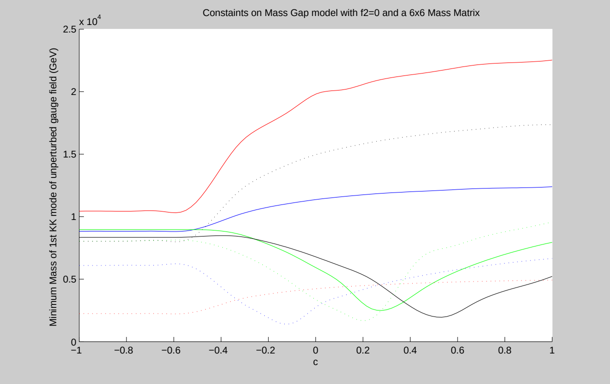
- Introduced as 5D approximation of Klebanov-Strassler solution. I.e. no IR cut off but deformed tip of the cone.
- Described by;

$$ds^2 = h^{-\frac{1}{2}}(r)\eta^{\mu\nu}dx_\mu dx_\nu - h^{\frac{1}{2}}(r)dr^2 \quad \text{with} \quad h(r) = \frac{R^4}{R'^4 + \mathrm{f}_2 R^2 r^2 + r^4}. \qquad (11)$$

- Where $0 \leqslant r \leqslant R$. Randall and Sundrum described by $h(r) = \frac{R^4}{r^4}$ but cut off at $r = R'$ but essentially very similar to Mass Gap Metric.
- Turns out EW constraints are minimal when $f_2 = 0$.

EW constraints from a Mass Gap Metric

For a value $f_2 = 0$ the constraints given by;



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Summary and Conclusion

- Whether the little hierarchy problem is a problem is essentially a matter of opinion, however with the LHC approaching we do need to know if it is possible to directly observe extra dimensions and conversely if we do see a W' or a Z' we need to know what models could fit such an observation.
- Considering the effects of the higher KK modes appears to raise the lower bound in the case of the RS model. E.g. $\gtrsim 27$ TeV to $\gtrsim 31$ TeV.
- A small change in geometry does appear to have a considerable effect on constraints. E.g. $\gtrsim 14.5$ TeV to $\gtrsim 10.5$ TeV.
- This is very much work in progress. Many questions still to be answered such as the effect of custodial symmetry or the going to 10D.